2.1: Instantaneous Rate of Change

Definition: The **instantaneous rate of change** of f at a, is defined to be the limit of the average rates of change of f over shorter and shorter time intervals around a. We can write this mathematically as

$$\lim_{\substack{b \to a \\ \text{“}b \text{ approaches } a\text{”}}} \frac{f(b) - f(a)}{b - a}.$$

Definition: The **derivative** of f at a, written $f'(a)$, is defined to be the instantaneous rate of change of f at the point a. It is common to write the derivative mathematically as

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Question: Explain in your own words how the two limits above actually represent the same thing, i.e., explain how they are equal.

Remark: It is common that we will see

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

but this should not confuse you at this point since the variable x (and h) above is (are) just “dummy variables.”

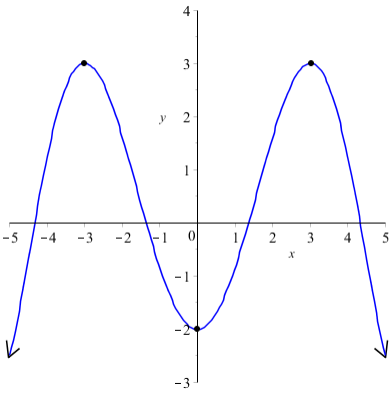
Exercise 1: In a time of t seconds, a particle moves a distance of s meters from its starting point, where $s = 4t^2 + 3$.

(a) Find the average velocity between $t = 1$ and $t = 1 + h$ if:

$$(i) \quad h = 0.1, \quad (ii) \quad h = 0.01, \quad (iii) \quad h = 0.001.$$

(b) Use your answers to part (a) to estimate the instantaneous velocity of the particle at time $t = 1$.

Exercise 2: Consider the function f given by the graph below. For each pair of numbers, determine which is larger.



(a) $f(3)$ or $f(4)$

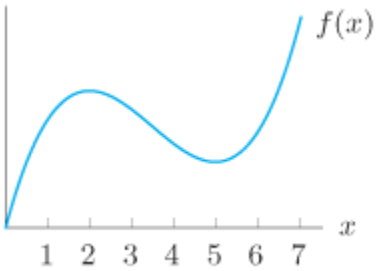
(b) $f(4) - f(3)$ or $f(4) - f(2)$

(c) $\frac{f(4)-f(3)}{4-3}$ or $\frac{f(4)-f(2)}{4-2}$

(d) $f'(3)$ or $f'(4)$

Exercise 3: The graph of a function $y = f(x)$ is shown below. Indicate whether each of the following quantities is positive or negative and illustrate your answers graphically.

(a) $f'(1)$ (b) $\dfrac{f(3) - f(1)}{3 - 1}$ (c) $f(4) - f(2)$



Exercise 4: For the function below, at what labeled points is the slope of the graph positive? Negative? At which labeled point does the graph have the greatest slope? The least slope?